

Electron-fluctuation interaction in a non-Fermi superconductor

M. Crisan

Department of Theoretical Physics, University of Cluj, 3400 Cluj, Romania

C. P. Moca

Department of Physics, University of Oradea, 3700 Oradea, Romania

I. Tifrea

Department of Theoretical Physics, University of Cluj, 3400 Cluj, Romania

Abstract

We studied the influence of the amplitude fluctuations of a non-Fermi superconductor on the energy spectrum of the 2D Anderson non-Fermi system. The classical fluctuations give a temperature dependence in the pseudogap induced in the fermionic excitations.

INTRODUCTION

The microscopic description of the superconducting state in cuprate materials is a very difficult problem because at the present time is generally accepted that in the normal state the elementary excitations are not described by the Fermi liquid theory. However, using the BCS-like pairing model the Gorkov equations have been applied to describe the superconducting state in the hypothesis that the normal state is a non-Fermi liquid described by the Anderson model¹. The superconducting state properties have been discussed by different authors^{2–8} and even if these descriptions are phenomenological, they can be a valid starting point for a microscopic model. Recent experimental data (ARPES) showed that these materials present even more remarkable deviations from the Fermi liquid behavior due to the occurrence of the pseudogap at the Fermi surface.

The occurrence of the pseudogap has been explained using different concepts as: the spin fluctuations⁹, preformed pairs¹⁰, SO(5) symmetry¹¹, spin-charge separation¹², the fluctuations of the order parameter induced pseudogap¹³.

In this paper we start with a non-Fermi liquid description of the superconducting state (See Ref. 1–8) and consider the interaction between the order parameter fluctuations and the electrons (Section 2 and Section 3). This problem has been studied by Abrahams et al.¹⁴, Marcelja¹⁵ and Schmid¹⁶ for BCS superconductors and the theory explained the tunneling experiments on films, by the modification of the density of states by a pseudogap which appears at a temperature higher than the BCS critical temperature. Using such an approximation we will calculate (Section 4) the pseudogap due to the electron-fluctuation interaction and in the simple mode-mode approximation the temperature dependence of it

will be obtained.

Finally (Section 5) we compare our results with the other theoretical models for the cuprate superconductors.

THE MODEL

The non-Fermi behavior of the normal state for the cuprate superconductors proposed by Anderson¹ was developed by different authors²⁻⁸ in order to describe the superconducting state in the framework of the BCS theory. In the normal state the electrons are described by the Green's function

$$G_0(\mathbf{k}, i\omega_n) = \frac{\omega_c^{-\alpha}}{(i\omega_n - \varepsilon_{\mathbf{k}})^{1-\alpha}} \quad (1)$$

where ω_c is a cutoff energy and $0 < \alpha < 1$.

In the following we consider that the superconducting state appears due an attractive interaction and is described by the BCS like order parameter $\Delta_{\mathbf{k}}$ which can be calculated from the Gorkov equations. The fluctuations of this parameter can interact with the electrons and the fermionic spectrum of the elementary excitations changes. Such an effect has been studied in the BCS superconductors by different authors¹³⁻¹⁵ and it was showed that this interaction gives a contribution to the density of states for $T > T_c$ which explained the behavior of the tunneling measurements.

For a superconductor described by the Gorkov like equations with the normal state described by Eq. (1) the propagator of the fluctuations has the expression:

$$D(\mathbf{q}, i\omega_n) = \frac{1}{V^{-1} + \Pi(\mathbf{q}, i\omega_n)} \quad (2)$$

where V is the attractive interaction between the electrons and $\Pi(\mathbf{q}, i\omega_n)$ is the polarization operator defined as

$$\Pi(\mathbf{q}, i\omega_n) = T \sum_{\omega_l} \int \frac{d\mathbf{p}}{(2\pi)^2} G(\mathbf{p}, i\omega_l) G(\mathbf{q} - \mathbf{p}, i\omega_n - i\omega_l) \quad (3)$$

where $G(\mathbf{p}, i\omega_l)$ is the Green's function related to electrons, which in terms of a Dyson equation has the following form

$$G^{-1}(\mathbf{p}, i\omega_l) = G_0^{-1}(\mathbf{p}, i\omega_l) - \Sigma(\mathbf{p}, i\omega_l) \quad (4)$$

where the self energy is given by

$$\Sigma(\mathbf{p}, i\omega_l) = -T \sum_{\omega_n} \int \frac{d\mathbf{q}}{(2\pi)^2} D(\mathbf{q}, i\omega_n) G(\mathbf{q} - \mathbf{p}, i\omega_n - i\omega_l) \quad (5)$$

Eqs. (2-5) have to be solved self consistent, but this cannot be done analytically. However, in the mode-coupling approximation it can be done and we can calculate the new energy of the electronic excitations.

MODE-COUPPLING APPROXIMATION

In this approximation we consider first that $G(\mathbf{k}, i\omega_n) \approx G_0(\mathbf{k}, i\omega_n)$ and from Eq. (3) we define the polarization

$$\Pi_0(\mathbf{q}, i\omega_m) = \int \frac{d\mathbf{k}}{(2\pi)^2} S(\mathbf{k}, \mathbf{q}, i\omega_m) \quad (6)$$

where

$$S(\mathbf{k}, \mathbf{q}, i\omega_m) = (-1)^{1-\alpha} T \sum_{\omega_n} \frac{\omega_c^{-\alpha}}{(i\omega_n - \varepsilon_{\mathbf{k}})^{1-\alpha} (i\omega_n - i\omega_m + \varepsilon_{\mathbf{q}-\mathbf{k}})^{1-\alpha}} \quad (7)$$

We performed the analytical calculation of $\Pi_0(\mathbf{q}, i\omega_m)$ given by Eq. (6) (See Appendix) and from Eq. (2) the propagator for the order parameter fluctuations has been obtained as

$$D_0^{-1}(\mathbf{q}, i\omega_n) = N(0)A(\alpha) \left\{ C(\alpha) \left[\left(\frac{T}{\omega_c} \right)^{2\alpha} - \left(\frac{T_c}{\omega_c} \right)^{2\alpha} \right] + \frac{i\omega_n(1-\alpha)}{T} M \left(\alpha, \frac{T}{\omega_c}, \frac{\omega_D}{\omega_c} \right) + \left(\frac{v_F q}{2T} \right)^2 (1-\alpha)^2 N \left(\alpha, \frac{T}{\omega_c}, \frac{\omega_D}{\omega_c} \right) \right\} \quad (8)$$

where the critical temperature T_c has been obtained^{7,8} as

$$T_c^{2\alpha} = \frac{1}{C(\alpha)} \left[D(\alpha) \omega_D^{2\alpha} - \frac{\omega_c^{2\alpha}}{A(\alpha)N(0)V} \right] \quad (9)$$

and the constants from Eqs. (8) and (9) are

$$A(\alpha) = \frac{2^{2\alpha}}{\pi} \sin \pi(1-\alpha)$$

$$C(\alpha) = \Gamma^2(\alpha) [1 - 2^{1-2\alpha}] \zeta(2\alpha)$$

$$D(\alpha) = \frac{\Gamma(1-2\alpha)\Gamma(\alpha)}{2\alpha\Gamma(1-\alpha)}$$

$$\begin{aligned} M \left(\alpha, \frac{T}{\omega_c}, \frac{\omega_D}{\omega_c} \right) &= \frac{\Gamma(\alpha-1)\Gamma(\alpha-1/2)}{2\sqrt{\pi}} [1 - 2^{2-2\alpha}] \zeta(2\alpha-1) \left(\frac{T}{\omega_c} \right)^{2\alpha} \\ &\quad - \frac{B(3-2\alpha, \alpha-2)}{4(2\alpha-2)} \left(\frac{\omega_D}{\omega_c} \right)^{2\alpha-2} \left(\frac{T}{\omega_c} \right)^2 \\ N \left(\alpha, \frac{T}{\omega_c}, \frac{\omega_D}{\omega_c} \right) &= \left[\frac{2\Gamma(\alpha-2)\Gamma(\alpha-1/2)}{\sqrt{\pi}} + \Gamma^2(\alpha) \right] \frac{1-2^{3-2\alpha}}{4} \zeta(2-2\alpha) \left(\frac{T}{\omega_c} \right)^{2\alpha} \\ &\quad - \frac{B(3-2\alpha, \alpha-2)}{4(2\alpha-2)} \left(\frac{\omega_D}{\omega_c} \right)^{2\alpha-2} \left(\frac{T}{\omega_c} \right)^2 \end{aligned} \quad (10)$$

$B(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$ and $\Gamma(x)$ are the Euler' functions and $\zeta(x)$ is the Riemann function.

Using a similar form with the one introduced by Schmid the fluctuation propagator will be written as

$$D_0^{-1}(\mathbf{q}, i\omega_n) = N(0) [b(\alpha)\tau(\alpha) + ia(\alpha)\omega_n + \xi^2(\alpha, T)q^2] \quad (11)$$

where

$$\tau(\alpha) = \left(\frac{T}{T_c}\right)^{2\alpha} - 1 \quad (12)$$

$$a(\alpha) = \frac{M(\alpha, T/\omega_c, \omega_D/\omega_c)}{T} (1 - \alpha) A(\alpha) \quad (13)$$

$$b(\alpha) = A(\alpha) C(\alpha) \left(\frac{T_c}{\omega_c}\right)^{2\alpha} \quad (14)$$

and

$$\xi(\alpha) = \frac{v_F^2 (1 - \alpha)^2}{4T^2} N\left(\alpha, \frac{T}{\omega_D}, \frac{\omega_D}{\omega_c}\right) A(\alpha) \quad (15)$$

In the approximation $\Sigma \ll \pi T$ the Green function given by Eq. (4) will be approximated as $G = G_0 + G_0 \Sigma G_0$ and Π will be modified by $\delta\Pi$ also linear in Σ . Following Ref. 11 we calculated $\delta\Pi$ in the "box approximation" as

$$\delta\Pi = 2T^2 \sum_n \int \frac{d\mathbf{p}}{(2\pi)^2} G_0^2(\mathbf{p}, i\omega_n) G_0^2(-\mathbf{p}, -i\omega_n) \int \frac{d\mathbf{q}}{(2\pi)^2} D(\mathbf{q}, \omega_n = 0) \quad (16)$$

where

$$D^{-1}(\mathbf{q}, i\omega_n) = \frac{1}{V} + \Pi(\mathbf{q}, i\omega_n) + \delta\Pi(\mathbf{q}, i\omega_n) \quad (17)$$

In order to calculate $\delta\Pi$ we introduce

$$B_0 = \frac{1}{N(0)} T \sum_n \int \frac{d\mathbf{p}}{(2\pi)^2} G_0^2(\mathbf{p}, i\omega_n) G_0^2(-\mathbf{p}, -i\omega_n) \quad (18)$$

where $N(0) = m/2\pi$. If we use for the electronic Green function Eq. (1) we obtained

$$B_0(T) = \frac{B(1/2, 3/2 - 2\alpha)}{\pi} \frac{[2^{3-4\alpha} - 1] \zeta(3 - 4\alpha)}{2^{3-4\alpha}} \frac{\omega_c^{-4\alpha}}{(\pi T)^{2-4\alpha}} \quad (19)$$

If we introduce $\tilde{\tau}(\alpha) = \tau(\alpha) + \delta\Pi/N(0)$ the fluctuation propagator given by Eq. (17) will be

$$D^{-1}(\mathbf{q}, i\omega_n) = b(\alpha)\tilde{\tau}(\alpha) + ia(\alpha)\omega_n + \xi^2(\alpha, T)q^2 \quad (20)$$

where

$$\tilde{\tau}(\alpha) - \tau(\alpha) = \frac{2B_0(T)}{N(0)}T \int \frac{\mathbf{q}}{(2\pi)^2} \frac{1}{\tilde{\tau}(\alpha) + \xi^2(\alpha, T)q^2} \quad (21)$$

If we perform this integral taking the upper limit $q_M = 1/\xi(\alpha, T)$ from Eq. (21) we get

$$\tilde{\tau}(\alpha) - \tau(\alpha) = \frac{B_0(T)T}{2\pi N(0)\xi^2(\alpha, T)} \ln \frac{1 + \tilde{\tau}(\alpha)}{\xi^2(\alpha, T)} \quad (22)$$

For realistic parameters ($T_c = 100K$, $\omega_c = 200K$) the difference $\tilde{\tau}(\alpha) - \tau(\alpha)$ becomes important only near a critical value of α defined by $\xi(\alpha_c) = 0$. In the BCS limit ($\alpha = 0$) this parameter is small and this behavior can be associated with the occurrence of the preformed pairs in the domain $T_c < T < T^*$, controlled by α . This behavior is in fact due to the occurrence of a pseudogap in the electronic excitations.

ELECTRONIC SELF-ENERGY

The self-energy due to the interaction between electrons and fluctuations is given by Eq. (5) where $D(\mathbf{q}, i\omega_n)$ is given by Eq. (20). First we calculate the summation over the Matsubara frequencies ω_n

$$S = T \sum_n D(i\omega_n)G(i\omega_n - i\omega_l) = T \sum_n \frac{(-1)^\alpha \omega_c^{-\alpha}}{N(0)(b\tilde{\tau} + ia\omega_n + \xi^2 q^2)(i\omega_l + \varepsilon_{\mathbf{k}} - i\omega_n)^{1-\alpha}} \quad (23)$$

transforming this sum in a contour integral which has a pole at $\Omega(\mathbf{q}) = -(b\tilde{\tau} + \xi^2 q^2)/a$ and a cut line from $\varepsilon_{\mathbf{k}} + i\omega_l$ to ∞ in the upper semiplane. From Eq. (13) we can see that $a(\alpha) = -|a(\alpha)|$ and in fact $\Omega(\mathbf{q}) = (b\tilde{\tau} + \xi^2 q^2)/|a|$. Performing this integral we obtain

$$S = \frac{\omega_c^{-\alpha}}{N(0)} \frac{n(\Omega(\mathbf{q}))}{(-i\omega_l - \varepsilon_{\mathbf{k}} - \Omega(\mathbf{q}))^{1-\alpha}} - \frac{\omega_c^{-\alpha}}{N(0)} \frac{\sin[\pi(1-\alpha)]}{\pi} \int_{\varepsilon_{\mathbf{k}}}^{\infty} dt \frac{f(t)}{(b\tilde{\tau} - |a|(t + i\omega_l) + \xi^2 q^2)(t - \varepsilon_{\mathbf{k}})^{1-\alpha}} \quad (24)$$

where $n(x)$ is the Bose-Einstein function and $f(x)$ is the Fermi-Dirac function and $\varepsilon_{\mathbf{k}} = k^2/2m - E_F$. The integral from the second contribution in Eq. (24) will be performed using the expansion $f(t) = \sum_{m=0} (-1)^m \exp[-\beta(m+1)t]$ and the last term becomes

$$I_1 = \sum_{m=0}^{\infty} \frac{(-1)^m}{|a|} \frac{(\varepsilon_{\mathbf{k}} + \Omega(\mathbf{q}))^{\alpha/2+1}}{[\beta(m+1)]^{\alpha/2}} \exp\left[\frac{\beta(m+1)(\Omega(\mathbf{q}) - \varepsilon_{\mathbf{k}})}{2}\right] \times \Gamma(\alpha) W_{-\alpha/2, \alpha/2-1/2}[\beta(m+1)(\Omega(\mathbf{q}) + \varepsilon_{\mathbf{k}})] \quad (25)$$

where the Whittaker function $W_{\lambda, \mu}(z)$ will be approximated as $W_{\lambda, \mu} \cong e^{-z/2} z^{\lambda}$. This results give for Eq. (24) the expression

$$S = \frac{\omega_c^{-\alpha}}{N(0)} \frac{n(\Omega(\mathbf{q}))}{(-i\omega_l - \varepsilon_{\mathbf{k}} - \Omega(\mathbf{q}))^{1-\alpha}} + \frac{\omega_c^{-\alpha}}{N(0)} \frac{\sin[\pi(1-\alpha)]}{\pi} \sum_{m=0}^{\infty} \frac{(-1)^m}{|a|} \frac{\varepsilon_{\mathbf{k}} + i\omega_l + \Omega(\mathbf{q})}{[\beta(m+1)]^{\alpha}} \Gamma(\alpha) \exp[\beta(m+1)\varepsilon_{\mathbf{k}}] \quad (26)$$

In the limit $k \cong k_F$ the second term denoted by S_2 becomes

$$S_2 = \frac{\omega_c^{-\alpha}}{N(0)} \frac{\sin[(1-\alpha)\pi]}{\pi} \frac{i\omega_l |a| + b\tilde{\tau} + \xi^2 q^2}{|a|^2} \Gamma(\alpha) [1 - 2^{1-\alpha}] \zeta(\alpha) \quad (27)$$

and if $T \rightarrow T_c$, $\omega_l \rightarrow 0$ and $q \rightarrow 0$ this term can be neglected. This approximation is in fact equivalent with the physical picture proposed by Vilk and Tremblay¹³ in which the occurrence of the pseudogap is given by the interaction between the electrons and the classical fluctuations. Indeed, in this regime the first term of Eq. (26) can be written as

$$S \cong \frac{1}{N(0)} n(\Omega(\mathbf{q})) G(\mathbf{k}, -i\omega_l + \Omega(\mathbf{q})) \quad (28)$$

and the electronic self-energy becomes

$$\Sigma(\mathbf{p}, \omega + i0) \cong -\Delta_{pg}^2 G(\mathbf{k}, -i\omega_l) \quad (29)$$

where we considered $\varepsilon_{\mathbf{k}} \gg \Omega(\mathbf{q})$ and

$$\Delta_{pg}^2 = \frac{1}{N(0)|a|} \int \frac{d\mathbf{q}}{(2\pi)^2} n(\Omega(\mathbf{q})) \quad (30)$$

will be approximated as

$$\Delta_{pg}^2(T) \cong \frac{T}{4\pi|a|N(0)} \int_0^{q_M} \frac{q dq}{(\tilde{\tau} + \xi^2 q^2)/|a|} \quad (31)$$

where q_M is the wave number cutoff. From Eq. (31) we calculate the temperature dependence of $\Delta_{pg}(T)$ as

$$\Delta_{pg}^2(T) = \frac{T}{4\pi N(0)\xi^2} \ln \left(1 + \frac{\xi^2}{\tilde{\tau}} q_M^2 \right) \quad (32)$$

DISCUSSIONS

We showed that a temperature dependent pseudogap appears in a non-Fermi superconductor due to the interaction between electrons and the fluctuations of the order parameter amplitude.

The mode-mode coupling, valid in the weak coupling approximation, can give relevant results, even for the intermediate coupling studied by the Levin group¹⁷ using the resonant scattering model. The method, recently applied by Norman et al¹⁸, Randeria¹⁹ can be applied for the spin-fluctuation model proposed by Chubukov⁹ in order to study the temperature dependence of the pseudogap. In Ref. 18 and 19 the filling in of the pseudogaps due to the increment of the temperature is given by the broadening in the self-energy and is proportional to $T - T_c$. A similar broadening effect, proportional to $\tilde{\tau}(\alpha)$ was obtained in

our model and this can be seen very easy from Eq. (27) if in the electronic Green function we take the limit $q = 0$.

Recently such a model, for a Fermi liquid superconductor has been studied by Kristoffel and Ord²⁰ and their temperature dependence is different from our result. However, we mention that according to their model these authors have to obtain a result similar to the result given in¹³. The difference is given by the method of performing the integral over \mathbf{q} which is not correct in¹⁹.

Recently, Preotsi et al²¹ generalized the method given in¹³ taking into consideration the anysotropy in the dynamic susceptibility due to the interplane pairing.

I. ACKNOWLEDGMENTS

We thank Mohit Randeria and Andre-Marie Tremblay for useful discussions about recent developments in pseudogap models. The work was supported by MEI under the grant nr. 184/1998.

APPENDIX

The polarization $\Pi(\mathbf{q}, i\omega_m)$ for a 2D non-Fermi liquid is defined as

$$\begin{aligned}\Pi_0(\mathbf{q}, i\omega_m) &= T \sum_n \int \frac{d\mathbf{k}}{(2\pi)^2} G_0(\mathbf{k}, i\omega_n) G_0(\mathbf{q} - \mathbf{k}, i\omega_m - i\omega_n) \\ &= T \sum_n \int \frac{d\mathbf{k}}{(2\pi)^2} \frac{\omega_c^{-2\alpha}}{(i\omega_n - \varepsilon_{\mathbf{k}})^{1-\alpha} (i\omega_m - i\omega_n - \varepsilon_{\mathbf{q}-\mathbf{k}})^{1-\alpha}}\end{aligned}\tag{33}$$

which can be written as

$$\Pi_0(\mathbf{q}, i\omega_m) = \int \frac{d\mathbf{k}}{(2\pi)^2} S(\mathbf{k}, \mathbf{q}, i\omega_m)\tag{34}$$

where

$$S(\mathbf{k}, \mathbf{q}, i\omega_m) = (-1)^{1-\alpha} T \sum_n \frac{\omega_c^{-2\alpha}}{(i\omega_n - \varepsilon_{\mathbf{k}})^{1-\alpha} (i\omega_n + \varepsilon_{\mathbf{q}-\mathbf{k}} - i\omega_m)^{1-\alpha}} \quad (35)$$

In order to perform the summation in Eq. (35) we transform the summation in a contour integral

$$S(\mathbf{k}, \mathbf{q}, z) = - \oint_C \frac{dz}{2\pi i} n(z) F(z) \quad (36)$$

where $n(z)$ is the Fermi function and $F(z)$ is given by

$$F(z) = \frac{(-1)^{1-\alpha} \omega_c^{-2\alpha}}{(z - \varepsilon_{\mathbf{k}})^{1-\alpha} (z + \varepsilon_{\mathbf{q}-\mathbf{k}} - i\omega_m)^{1-\alpha}} \quad (37)$$

and the contour C is taken as $(-\infty, i\omega_n + \varepsilon_{\mathbf{q}-\mathbf{k}}) \cup (\varepsilon_{\mathbf{k}}, \infty)$. The integral in Eq. (36) has been evaluated as

$$\begin{aligned} \oint_C \frac{dz}{2\pi i} n(z) F(z) &= \frac{1}{2\pi i} \left\{ \int_{-\infty}^{-\varepsilon_{\mathbf{q}-\mathbf{k}}} dx n(x + i\omega_m) \frac{2i\omega_c^{-2\alpha} \sin[\pi(1-\alpha)]}{(x - \varepsilon_{\mathbf{k}})^{1-\alpha} (x + i\omega_n + \varepsilon_{\mathbf{q}-\mathbf{k}})^{1-\alpha}} \right. \\ &\quad \left. - \int_{\varepsilon_{\mathbf{k}}}^{\infty} dx n(x) \frac{2i\omega_c^{-2\alpha} \sin[\pi(1-\alpha)]}{(x - \varepsilon_{\mathbf{k}})^{1-\alpha} (x + i\omega_m + \varepsilon_{\mathbf{q}-\mathbf{k}})^{1-\alpha}} \right\} \end{aligned} \quad (38)$$

In order to perform the integral over x we express the dominators from (38) as

$$\begin{aligned} (x - \varepsilon_{\mathbf{k}})^{\alpha-1} (x + \varepsilon_{\mathbf{q}-\mathbf{k}} + i\omega_n)^{\alpha-1} &= (x - \varepsilon_{\mathbf{k}})^{\alpha-1} (x + \varepsilon_{\mathbf{k}})^{\alpha-1} \\ &\quad - (\alpha-1)(v_F q \cos \theta - i\omega_n)(x - \varepsilon_{\mathbf{k}})^{\alpha-1} (x + \varepsilon_{\mathbf{k}})^{\alpha-2} \\ &\quad + \frac{(\alpha-1)^2}{2} (v_F q \cos \theta)^2 (x - \varepsilon_{\mathbf{k}})^{\alpha-1} (x + \varepsilon_{\mathbf{k}})^{\alpha-3} \end{aligned} \quad (39)$$

and take for the Fermi function the expansion

$$n(x) = \sum_{m=0}^{\infty} (-1)^m \exp[-\beta(m+1)x] \quad (40)$$

Using now the integrals

$$\int_u^\infty dx (x + \beta)^{-\nu} (x - u)^{\mu-1} = (u + \beta)^{\mu-\nu} B(\nu - \mu, \mu) \quad (41)$$

$$\begin{aligned} \int_u^\infty dx (x + \beta)^{2\nu-1} (x - u)^{2\rho-1} \exp[-\mu x] &= \frac{(u + \beta)^{\nu+\rho+1}}{\mu^{\nu+\rho}} \exp\left[\frac{(\beta - u)\mu}{2}\right] \\ &\times \Gamma(2\rho) W_{\nu-\rho, \nu+\rho-1/2}(u\mu + \beta\mu) \end{aligned} \quad (42)$$

we calculated $S(\mathbf{k}, \mathbf{q}, i\omega_m)$ as

$$\begin{aligned} S(\mathbf{k}, \mathbf{q}, \omega_m) &= \frac{\omega_c^{-2\alpha} \sin[\pi(1 - \alpha)]}{\pi} \\ &\times \left\{ -(2\varepsilon_{\mathbf{k}})^{2\alpha-1} B(1 - 2\alpha, \alpha) \right. \\ &+ \frac{2\Gamma(\alpha)}{\sqrt{\pi}} \sum_{m=0}^{\infty} (-1)^m \frac{(2\varepsilon_{\mathbf{k}})^{\alpha-1/2}}{[\beta(m+1)]^{\alpha-1/2}} K_{\alpha-1/2}[\varepsilon_{\mathbf{k}}\beta(m+1)] \\ &+ (\alpha - 1)(v_F q \cos \theta - \omega_m) \left[-(2\varepsilon_{\mathbf{k}})^{2\alpha-2} B(2 - 2\alpha, \alpha - 1) \right. \\ &+ \left. \frac{\Gamma(\alpha - 1)}{\sqrt{\pi}} \sum_{m=0}^{\infty} (-1)^m \frac{(2\varepsilon_{\mathbf{k}})^{\alpha-1/2}}{[\beta(m+1)]^{\alpha-3/2}} K_{\alpha-3/2}[\varepsilon_{\mathbf{k}}\beta(m+1)] \right] \\ &+ \frac{(\alpha - 1)^2 (v_F q \cos \theta)^2}{2} \left[-(2\varepsilon_{\mathbf{k}})^{2\alpha-3} B(3 - 2\alpha, \alpha - 2) \right. \\ &+ \frac{\Gamma(\alpha - 2)}{\sqrt{\pi}} \sum_{m=0}^{\infty} (-1)^m \frac{(2\varepsilon_{\mathbf{k}})^{\alpha-1/2}}{[\beta(m+1)]^{\alpha-5/2}} K_{\alpha-3/2}[\varepsilon_{\mathbf{k}}\beta(m+1)] \\ &+ \left. \left. \frac{2\Gamma(\alpha - 1)}{\sqrt{\pi}} \sum_{m=0}^{\infty} (-1)^m \frac{(2\varepsilon_{\mathbf{k}})^{\alpha-3/2}}{[\beta(m+1)]^{\alpha-3/2}} K_{\alpha-3/2}[\varepsilon_{\mathbf{k}}\beta(m+1)] \right] \right\} \end{aligned} \quad (43)$$

Eq. (34) will be written as

$$\Pi(\mathbf{q}, i\omega_m) = 2N(0) \int_0^{2\pi} \frac{d\theta}{2\pi} \int_0^{\omega_D} d\varepsilon S(\varepsilon, \mathbf{q}, i\omega_m) \quad (44)$$

and using the relation

$$\int_0^\infty x^\mu K_\nu(ax) dx = 2^{\mu-1} a^{-\mu-1} \Gamma\left(\frac{1+\mu+\nu}{2}\right) \Gamma\left(\frac{1+\mu-\nu}{2}\right)$$

we obtained

$$\Pi_0(\mathbf{q}, \omega) = \frac{2N_0 \sin[\pi(1 - \alpha)]}{\pi}$$

$$\begin{aligned}
& \times \left\{ -\frac{2^{2\alpha-1}B(1-2\alpha, \alpha)}{2\alpha} \left(\frac{\omega_D}{\omega_c}\right)^{2\alpha} + \Gamma^2(\alpha) \frac{1-2^{1-2\alpha}}{2^{1-2\alpha}} \zeta(2\alpha) \left(\frac{T}{\omega_c}\right)^{2\alpha} \right. \\
& + \frac{i\omega(1-\alpha)}{\omega_c} \left[-\frac{2^{2\alpha-2}B(2-2\alpha, \alpha-1)}{2\alpha-1} \left(\frac{\omega_D}{\omega_c}\right)^{2\alpha-1} \right. \\
& + \frac{\Gamma(\alpha-1)\Gamma(\alpha-1/2)}{\sqrt{\pi}} \frac{1-2^{2-2\alpha}}{2^{2-2\alpha}} \zeta(2\alpha-1) \left(\frac{T}{\omega_c}\right)^{2\alpha-1} \left. \right] \\
& + \frac{(v_F q)^2(1-\alpha)^2}{4\omega_c^2} \left[-\frac{2^{2\alpha-3}B(3-2\alpha, \alpha-2)}{2\alpha-2} \left(\frac{\omega_D}{\omega_c}\right)^{2\alpha-2} \right. \\
& + \left. \left(\frac{2\Gamma(\alpha-2)\Gamma(\alpha-1/2)}{\sqrt{\pi}} + \Gamma^2(\alpha-1) \right) \frac{1-2^{3-2\alpha}}{2^{3-2\alpha}} \zeta(2-2\alpha) \left(\frac{T}{\omega_c}\right)^{2\alpha-2} \right] \left. \right\} \quad (45)
\end{aligned}$$

Using now the Thouless criterion

$$1 + V \text{Re} \Pi(\mathbf{q} = 0, i\omega_m = 0) = 0 \quad (46)$$

we calculate the critical temperature

$$\begin{aligned}
\left(\frac{T_c}{\omega_c}\right)^{2\alpha} &= \frac{B(1-2\alpha, \alpha)}{2\alpha\Gamma^2(\alpha)(1-2^{1-2\alpha})\zeta(2\alpha)} \left(\frac{\omega_D}{\omega_c}\right)^{2\alpha} \\
&\quad - \frac{\pi}{N(0)V\Gamma^2(\alpha)2^{2\alpha}(1-2^{1-2\alpha})\zeta(2\alpha)\sin[\pi(1-\alpha)]} \quad (47)
\end{aligned}$$

which is identical to Eq. (8) if we introduce $\lambda = N(0)V$.

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